Decoherence and Matter Wave Interferometry

Tabish Qureshi*
Department of Physics, Jamia Millia Islamia, New Delhi-110025, India.

Anu Venugopalan†
Centre for Philosophy and Foundations of Science, Darshan Sadan, E-36, Panchsheel Park, New Delhi-110011, India. and School of Basic and Applied Sciences, G.G.S. Indraprastha University, Delhi - 110 006, India.

A two-slit interference of a massive particle in the presence of environment induced decoherence is theoretically analyzed. The Markovian Master equation, derived from coupling the particle to a harmonic-oscillator heat bath, is used to obtain exact solutions which show the existence of an interference pattern. Interestingly, decoherence does not affect the pattern, but only leads to a reduction in the fringe visibility.

PACS numbers: 03.65.Yz 03.75.Dg 03.65.Ud

I. INTRODUCTION

Recently it has been possible to observe diffraction of large molecules like C_{60}, which are expected to behave like classical particles [1]. Interference effects have also been observed in larger molecules [2]. These effects are a consequence of the linear superposition principle applied to the wave functions of the particles and can only be described quantum mechanically. For massive particles, the existence of such superpositions is classically unimaginable and is not a familiar observation in the real physical world . The quantum-classical transition and the nature of classicality as an emergent property of an underlying quantum system has been the subject of a lot of study in recent years [3, 4]. The decoherence phenomenon has been widely discussed and accepted as the mechanism responsible for the emergence of familiar classical features in the real physical world. Decoherence results from the irreversible coupling of the system to its environment. The recent experimental observation of diffraction and interference patterns for large molecules raises a natural question: How far can one go before decoherence effects destroy the interference pattern of massive objects? The effect of decoherence on matter wave interferometry has been explored quite well experimentally. Hornberger et al have studied the effect of decoherence due to collisions of the interfering particle with gas molecules [5]. They observed that with increasing pressure of the gas, the fringe visibility goes down. Chapman et al have studied the effect of photons scattering off interfering atoms [6]. Kokorowski et al have studied the same effect using multiple photons [7]. Sonnentag and Hasselbach have studied effect of decoherence in electron biprism interferometer [8]. On the theoretical side there have been several focussed studies on specific systems [9, 10]. However, there is a need to understand the effect of decoherence on the observed interference pattern in a double-slit experiment in a simplified way. It is intuitively obvious that the resulting interference pattern in such a scenario would be a consequence of the interplay between the strength of decoherence, the slit separation and the distance the particle travels from the slit to the screen. One would like to understand to what extent each of these parameters play a role.

Decoherence effects on quantum superpositions have generally been studied by assuming an initial state which is a superposition of two spatially localized wave-packets. For an initial state of the form $\psi(x) = \psi_1(x) + \psi_2(x)$, the density matrix in the position representation is

$$\begin{aligned}\rho(x,x') &= \psi_1(x)\psi_1^*(x') + \psi_2(x)\psi_2^*(x') \\ &\quad + \psi_1(x)\psi_2^*(x') + \psi_2(x)\psi_1^*(x'). \qquad (1)\end{aligned}$$

For localized wavepackets (which best describe a massive particle), the four terms in this density matrix correspond to four peaks. The last two terms correspond to the off-diagonal peaks. Decoherence arguments show that the off-diagonal peaks of the density matrix die out, in time, because of the effect of the environment [4]. Thus, the appearance of classical behaviour via decoherence is marked by the dynamical transition of the pure density matrix to a statistical mixture. For example, using the Markovian Master equations with some approximations, Zurek has argued that the density matrix for a free particle in an initial coherent superposition of two Gaussian wave packets separated by Δx decoheres (i.e. the off-diagonal peaks decay) over a time scale which goes inversely as the square of the separation, Δx^2, between the two parts of the superposition [4]. For classical systems and standard macroscopic separations, this decoherence time is shown to be almost 10^{-40} times smaller than the thermal relaxation time of the system. Macroscopic superpositions, are, thus, almost instantaneously reduced to a statistical mixture, a situation which is classically interpretable. The decoherence approach has been used to study many

*Electronic address: tabish@jamia-physics.net
†Electronic address: anu.venugopalan@gmail.com

models in the context of quantum measurement and the decoherence mechanism has been explored in the experimental regime also [11, 12]. It is now generally accepted that the two main signatures of the decoherence mechanism are, (i) in the classical regime decoherence takes place over a time scale that is much smaller than the thermal relaxation time of the system, and (ii) the decoherence time goes inversely as the square of the separation between the two parts of the superposition [4, 13]. If one were to look at a two-slit interference situation, one can write a state of the form $\psi(x) = \psi_1(x) + \psi_2(x)$, where $\psi_1(x)$ and $\psi_2(x)$ correspond to the probability amplitudes for the particle to pass through slit 1 and slit 2, respectively. As is well known, the interference pattern corresponds to the position probability distribution of the time evolved wave function:

$$\begin{aligned} |\psi(x,t)|^2 &= \psi_1(x,t)\psi_1^*(x,t) + \psi_2(x,t)\psi_2^*(x,t) \\ &\quad +\psi_1(x.t)\psi_2^*(x,t) + \psi_2(x,t)\psi_1^*(x,t) \\ &= \rho(x,x,t). \end{aligned} \tag{2}$$

One might be naively tempted to look at the off-diagonal components of the density matrix at the screen. However, the interference pattern is obtained from the probability distribution of of the particle on the screen, which is just the *diagonal* components of the density matrix. So, it is not obvious if the dying out of *off-diagonal* components of the density matrix, at the screen, as a consequence of decoherence also corresponds to a disappearance of the interference pattern. There is a need, therefore, to study the evolution of the state of the particle along with the effect of the environment, and analyze the emerging position probability distribution for the existence of an interference pattern. Savage and Walls have addressed this issue by studying the evolution of a superposition of two plane-waves under the influence of decoherence, and its effect on the interference [14]. However, though their results illustrate the effect of decoherence on the interference pattern, their use of plane waves to describe the state of the interfering molecules seems a little unrealistic as it is obvious that large molecules would be better described by localized wave-packets.

Recently, decoherence effects on two-slit diffraction has also been theoretically analyzed by treating the effect of the environment using certain phenomenological models [16, 17]. However, a fully quantum mechanical analysis, using a microscopic model of the environment has not yet addressed this issue. This kind of analysis is very relevant from the point of view of quantitatively probing the elusive quantum-classical boundary. The study is also highly relevant in the light of several recent proposals to exploit purely quantum mechanical features for quantum computation and quantum information processing [15].

In the following, we present an analysis of matter-wave interferometry in the presence of a dynamical, quantum environment. Starting with an initial superposition describing the double slit situation, we study the dynamics of the system through the Markovian master equation which takes into account the coupling to the environment. Exact solutions for the position probability distribution clearly bring out the role of environment-induced decoherence on the interference pattern. The rest of the paper is organized as follows. In Sec. II we set up the theoretical frame work for our analysis of decoherence and present our results. Further, we discuss fringe visibility and some of the factors affecting it. Finally, in Sec. III we summarize the main conclusions of this work.

II. THEORETICAL ANALYSIS

A. Coupling to the environment

The effect of decoherence on the quantum evolution of a system can be studied by coupling it to a model environment. A popular model for the environment is a set of non-interacting harmonic oscillators, which may arise out of different physical situations. The Hamiltonian for a "free" particle, coupled to such a model environment can be written as

$$H = \frac{p^2}{2m} + \sum_k \frac{P_k^2}{2M_k} + \frac{M_k\Omega_k}{2}\Big(X_k - \frac{C_k x}{M_k\Omega_k^2}\Big)^2. \tag{3}$$

Here x and p denote the position and momentum of the particle of mass m, and the second term represents the Hamiltonian for the bath of oscillators (environment). X_k and P_k are the position and momentum coordinates of the kth harmonic oscillator of the bath, C_ks are the coupling strengths and Ω_k's are the frequencies of the oscillators comprising the bath [18]. As one is interested in the dynamics of the particle, and not the detailed dynamics of the environment (which is not within one's control anyway), it is conventional to look at the reduced density matrix of the system, where environment variables have been traced over.

This Hamiltonian has been studied extensively, to obtain the dynamics of the reduced density matrix. Of particular relevance here is the case of "ohmic coupling", which gives the correct limit of classical dissipation. For our analysis, we deal directly with the Markovian master equation for the reduced density matrix of the system in the position representation, where the environmental degrees of freedom have been traced out[18, 19, 20]:

$$\begin{aligned} \frac{d\rho_r(x,x')}{dt} &= \frac{i\hbar}{2m}\left(\frac{\partial^2\rho_r}{\partial x^2} - \frac{\partial^2\rho_r}{\partial x'^2}\right) \\ &\quad -\gamma(x-x')\left(\frac{\partial\rho_r}{\partial x} - \frac{\partial\rho_r}{\partial x'}\right) - \frac{D}{4\hbar^2}(x-x')^2\rho_r. \end{aligned} \tag{4}$$

Here $D = 2m\gamma k_B T$ for a thermal bath. Equation (4) has been derived by assuming the harmonic oscillator environment to be in equilibrium, at a temperature T. The parameter γ can be assumed to signify the strength of the coupling of the particle to the environment - it has

its origin in the coupling strengths C_k appearing in the Hamiltonian (3).

At this stage it might be worthwhile to point out that this model of decoherence is fairly general, and spans a wide range of physical situations. For example, decoherence due to a photonic heat bath can be described by a particle interacting with the modes of the electromagnetic field, modeled by harmonic oscillators. In the case of intereference of electrons passing close to a conducting plate [8], the decoherence can be described by bosonic excitations of a Fermi sea of conduction electrons - this again, can be done easily by using a harmonic oscillator heat bath. In addition to this, the Master equation (4) can also be arrived at by studying the the quantum evolution of a particle undergoing collisions with smaller particles, using a stochastic formalism [20]. This relates very well to the experimental study of collisional decoherence in matter-wave interferometry [21].

B. Decoherence

Now that the framework of our analysis is set up, let us get back to matter wave interferometry. The particle encounters the double-slit, after which it travels a distance L, say, along the y-direction, before registering on the screen. Clearly, for the interference pattern to be visible, coherence along the x-direction is important, whereas the dynamics along the y-axis just serves to transport the particle from the slits to to the screen. In order to simplify the calculations, we assume that the particle emerges from the double-slit, travels along the y-axis with a well-defined average momentum p_0 and reaches the screen after a time $t_L = mL/p_0$. We now focus only on the time evolution of the particle's wave function in the x-direction.

At this stage, let us specify the functional form of the initial state that emerges from the double-slit. We assume that the action of the slit is to prepare a superposition of two Gaussian wave-packets, centered at the location of the respective slits, with a width equal to the width of the slit. We define the initial state as

$$\psi(x,0) \;=\; \frac{1}{\sqrt{2}}\frac{1}{(\pi/2)^{1/4}\sqrt{\epsilon}}\left(e^{-(x-x_0)^2/\epsilon^2} + e^{-(x+x_0)^2/\epsilon^2}\right), \tag{5}$$

where $x_0 = d/2$, d being the separation between the two slits. (5) represents two Gaussians centered at $x = \pm x_0$.

It may be noted that we have ignored the decoherence effects that could occur *before* the particle reaches the slit. From the point of view of an analytical calculation, it is difficult to take into account the effect of decoherence both before and after the double slit. Moreover, there could be specific experimental situations where decoherence occurs only after the double slit. In the experiment of Sonnentag and Hasselbach on interference of electrons, the decoherence takes place due to a metallic plate kept after the double-slit [8].

For the time evolution after the double slit, the initial density matrix for the particle state can be written using (5), as

$$\begin{aligned}\rho(R,r,0) \;=\; \frac{1}{\sqrt{2\pi\epsilon^2}}\Bigg(&e^{-\frac{(R-d)^2+r^2}{2\epsilon^2}} + e^{-\frac{(R+d)^2+r^2}{2\epsilon^2}} \\ &+e^{-\frac{(r-d)^2+R^2}{2\epsilon^2}} + e^{-\frac{(r+d)^2+R^2}{2\epsilon^2}}\Bigg),\end{aligned} \tag{6}$$

where $R = x + x'$ and $r = x - x'$ and d is the slit separation. In order to analyze interference, one needs to obtain the density matrix of the particle at a time t_L, when it reaches the screen. For this, we solve the master equation (4), with the initial condition (6). It turns out that an exact solution for $\rho_r(x,x,t_L)$ can be obtained. This will contain full information about the interference and the degree of decoherence. However, for this purpose we do not need the full density matrix of the particle - we are only interested in the position probability distribution of the particle on the screen, which is given by the diagonal part ($x = x'$) of the reduced density matrix. From the exact solutions one can see that the position probability distribution of the particle on the screen has the final form:

$$\begin{aligned}\rho_r(x,x,t_L) \;=\; \frac{1}{\sqrt{\pi}\Omega}\Bigg(&\frac{1}{2}e^{\frac{-(x-x_0)^2}{\Omega^2}} + \frac{1}{2}e^{\frac{-(x+x_0)^2}{\Omega^2}} + \\ &e^{-\frac{x^2+x_0^2}{\Omega^2}}e^{-\frac{\Gamma x_0^2}{\Omega^2}}\cos\left\{\frac{xd\hbar(1-e^{-2\gamma t_L})}{m\gamma\epsilon^2\Omega^2}\right\}\Bigg),\end{aligned} \tag{7}$$

where $\Gamma = \frac{D}{16m^2\epsilon^2\gamma^3}(4\gamma t_L + 4e^{-2\gamma t_L} - e^{-4\gamma t_L} - 3)$, $\Omega^2 = \epsilon^2 + \frac{\hbar^2(1-e^{-2\gamma t_L})^2}{m^2\epsilon^2\gamma^2} + \Gamma$.

Starting from (4), the result (7) is exact and represents the position probability distribution at the screen. The presence of the cosine term indicates the existence of the interference pattern. However, (7) is a very general result which can be used to study many things, like damped motion of the particle, the effect of friction and of course, decoherence. In order to relate it to decoherence, this result should be analyzed in a suitable limit. This limit is set by the requirement that the interaction of the particle with the environment be so weak that the dissipative effects are not noticeable, only the decoherence is. This will correspond to the limit $1/\gamma \gg t_L$, i.e, when the relaxation time of the system is much much larger than the time scales over which the experiment is performed and the observations made. In the following we make this approximation:

$$\begin{aligned}\Omega^2 \;&\approx\; \epsilon^2 + \left(\frac{2\hbar t}{m\epsilon}\right)^2 + \frac{2Dt_L^3}{3m^2} \approx \frac{\lambda_d^2 L^2}{\pi^2\epsilon^2}, \\ \Gamma \;&\approx\; \frac{2Dt_L^3}{3m^2\epsilon^2} = \frac{D\lambda_d^2 L^2}{6\pi^2\hbar^2}t_L.\end{aligned} \tag{8}$$

Here, we have assumed that $t_L = mL/p_0$, which implies that $\hbar t_L/m = \lambda_d L/2\pi$, λ_d being the de Broglie wavelength. In the expression for Ω^2, we have made a further

approximation, $\epsilon \ll \lambda_d L/\pi\epsilon$, i.e., the spread of the wave-packets, after travelling a distance L, is much larger than the original width. This will happen when the original width is very small. This is a realistic expectation since the slits should be narrow enough to let the two wave-packets spread and overlap with each other to lead to an interference pattern on the screen.

Using the approximations (8), (7) reduces to

$$\rho_r(x,x,t_L) = \frac{1}{\sqrt{\pi}\sigma}\left(\frac{1}{2}e^{\frac{-(x-x_0)^2}{\sigma^2}} + \frac{1}{2}e^{\frac{-(x+x_0)^2}{\sigma^2}} + e^{-\frac{x^2+x_0^2}{\sigma^2}}\exp(-\frac{t_L}{24\tau_D})\cos\left\{\frac{\pi x d}{\lambda_d L}.\right\}\right), \quad (9)$$

where $\sigma = \lambda_d L/\pi\epsilon$ and $\tau_D = \frac{\hbar^2}{2m\gamma k_B T d^2}$. Recalling earlier results on decoherence, one can recognize τ_D as the decoherence time of a superposition of two wave-packets, separated by a distance d, due to interaction with an environment at temperature T, with a coupling strength, or relaxation rate γ[4, 13].

Note that without the term $\exp(-\frac{t_L}{24\tau_D})$, (9) represents the interference (position probability distribution) of two wave-packets of initial width ϵ each. The expression also represents the interference pattern corresponding to a matter-wave of de-Broglie wavelength λ_d, having travelled a distance L from the double slits. The decoherence effects come in only through the term $\exp(-\frac{t_L}{24\tau_D})$. This term affords a simple physical meaning - two wave-packets, separated by the slit distance d lose their coherence after a time τ_D, which, by definition, is the decoherence time. If the wave-packets reach the screen at a time t_L which is much much larger than τ_D, no interference will be visible. For the interference pattern to be observable, τ_D should obviously be of the order of t_L.

Since our analysis is not tied to any one experimental situation, we use the following arbitrary values for various parameters: $d = 1$ μm, $\epsilon = 0.1$ μm, $\lambda_d = 5\times10^{-6}\mu$m, L = 20 cm. We assume that the Gaussian width of a wave-packet emerging out of a rectangular slit of width w will be roughly $w/2$. The dimensionless parameter t_L/τ_D is a measure of decoherence. Using these values, expression (9) is plotted in FIG. 1. Note that the overall Gaussian profile is due to the finite width of the slits. If the slits were infinitely narrow, one would see a flat profile with narrow interference peaks.

C. Fringe visibility

One quantity of particular importance in matter-wave interferometry is the fringe visibility, and the effect of decoherence on it. Conventionally, it is defined as

$$\mathcal{V} = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}, \quad (10)$$

where I_{max} and I_{min} represent the maximum and minimum intensity in neighbouring fringes, respectively. In

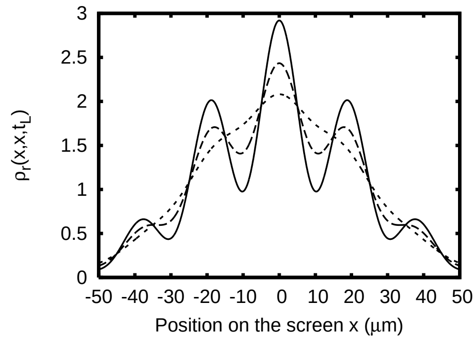

FIG. 1: The interference pattern $\rho_r(x,x,t_L)$ plotted against the position on the screen x, for $\epsilon = 0.1\mu m$, $\lambda_d = 5\times10^{-6}\mu$m, $L = 20$ cm, $d = 1.0\mu$m, and for various values of t_L/τ_D. Solid line represents $t_L/\tau_D = 4.0$, dashed line represents $t_L/\tau_D = 20$ and the dotted line represents $t_L/\tau_D = 60$.

reality, fringe visibility will depend on many things, including the width of the slits. For example, if the width of the slits is very large, the fringes may not be visible at all. If we focus on just the effect of decoherence on fringe visibility, we can assume that the slits are so narrow that we get an essentially flat background profile. This means that σ will be so large that the functions $e^{\frac{-(x\pm x_0)^2}{\sigma^2}}$ will have the same values at the points of maximum and minimum intensity. Maxima and minima of (9) will occur at points where the argument of the cosine is 1 and -1, respectively. Taking two such neighbouring points, the fringe visibility can be written as

$$\mathcal{V} = \frac{\exp(-\frac{t_L}{24\tau_D})}{\cosh(\frac{2x_n x_0}{\sigma^2})}, \quad (11)$$

where x_n denotes the mean position of nth fringe. Clearly, the fringe visibility goes down as t_L becomes larger than $\tau_D = \frac{\hbar^2}{2m\gamma k_B T d^2}$. This can most easily happen when either γ or m becomes large.

The expression for fringe visibility, (11), details its dependence on various physical parameters. It can be written in an expanded form as follows:

$$\mathcal{V} = \frac{\exp(-\frac{m\gamma k_B T d^2 t_L}{12\hbar^2})}{\cosh(\frac{2x_n x_0}{\sigma^2})}. \quad (12)$$

If the present microscopic model is compared with certain stochastic models of the environment, the parameter γ turns out to be proportional to the rate of collision of the interfering particle with the smaller particles constituting the environment [20]. If one tries to relate this model to collisional decoherence, as in the experiment of Hornberger et. al. [5], the pressure of the gas in the chamber should be directly proportional to the collision rate, and therefore to the parameter γ in our calculation. Eqn. (12) indicates an exponential decay of visibility with γ. This implies that the fringe visibility should go down exponentially with increasing pressure. This is in

broad agreement with the experimental findings of Hornberger et. al. [5], and does not depend on the fact that Hornberger et. al. use a Talbot-Lau interferometer instead of a double-slit. If one relates the calculation here to the experiment of Sonnentag and Hasselbach on interference of electrons, the decoherence is due to the electron's interaction with the metallic electrons of the plate. In this case, γ is given by [8]

$$\gamma = \frac{e^2\rho}{32\pi m z^3}, \tag{13}$$

where ρ is the resistivity of the place and z is the distance of the interfering beam from it. So, in this case too, the decoherence effects are expected to be stronger when electrons are closer to the plate, or z is smaller. This would mean, larger value of γ.

Fringe visibility is also very sensitive to the separation between the two slits, d. It decays exponentially with the square of the slit separation. To get an idea of the magnitude of the effect, for example, if a particular slit separation gives a visibility of 60 percent, doubling the slit separation will reduce the visibility to about 13 percent.

It is also clear from (12) that fringe visibility goes down exponentially with the mass of the interfering particle. So, if one were to use a molecule, say, twice as heavy as C_{60}, one would have to either reduce the temperature by the same factor, or decrease the pressure by the same factor, in order to get the same visibility as for C_{60}. This is in tune with the general expectation that the interference arising from the quantum nature of particles in the double-slit scenario analyzed here will be more vulnerable to decoherence for more massive particles. In many situations, the coupling to the environment may also be related to the mass, and such simplified logic may not always be correct.

III. CONCLUSION

We have theoretically analyzed the effect of decoherence on a massive particle, in a two-slit interference setup, using a quantum dynamical model of the environment. Interestingly, the interference pattern does not get distorted as a result of decoherence. The effect of decoherence is only to reduce the visibility of the interference fringes. The fringe visibility crucially depends on the decoherence time of a superposition of two freely evolving wave-packets, initially localized at the centers of the two slits. Our results clearly demonstrate the two main signatures of the decoherence mechanism, namely (a) the decoherence time is much smaller than the thermal relaxation time, and (b) the decoherence time is inversely proportional to the square of the "separation" between the two parts of the superposition. Our results also show that the fringe visibility goes down exponentially with the mass of the particle, with the temperature of the environment, with the square of the slit separation and with the coupling strength of the particle to the environment. Finally, the results are in good qualitative agreement with the matter-wave interferometry experiments carried out to study decoherence effects with large molecules and also with electrons.

[1] "**Wave particle duality of C60 molecules**", M. Arndt, O. Nairz, J. Vos-Andreae, C. Keller, G. van der Zouw and A. Zeilinger, *Nature* **401**, 680 (1999).

[2] "**Matter-wave interferometer for large molecules**", B. Brezger, L. Hackermller, S. Uttenthaler, J. Petschinka, M. Arndt, and A. Zeilinger, *Phys. Rev. Lett* **88**, 100404 (2002).

[3] "**Decoherence and the appearance of a classical world in quantum theory**", eds. D. Guilini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu and H. D. Zeh (Springer) (1996).

[4] "**Decoherence and the transition from quantum to classical**" W. H. Zurek, *Physics Today* **44** (10), 36 (1991); W. H. Zurek, Phys. Rev D **24**, 1516 (1981).

[5] "**Collisional decoherence observed in matter-wave interferometry**", K. Hornberger, S. Uttenthaler, B. Brezger,L. Hackermüller, M. Arndt, and A. Zeilinger, *Phys. Rev. Lett* **90**, 160401 (2003).

[6] "**Photon scattering from atoms in an atom interferometer: coherence lost and regained**", M. S. Chapman, T. D. Hammond, A. Lenef, J. Schmiedmayer, R. A. Rubenstein, E. Smith, and D. E. Pritchard, *Phys. Rev. Lett.* **75**, 3783 (1995).

[7] "**From single- to multiple-photon decoherence in an atom interferometer**," D. A. Kokorowski, A. D. Cronin, T. D. Roberts, and D. E. Pritchard, *Phys. Rev. Lett.* **86** 2191 (2001).

[8] "**Decoherence of Electron Waves Due to Induced Charges Moving Through a Nearby Resistive Material**," P. Sonnentag and F. Hasselbach *Braz. J. Phys.* **35**, 385 (2005).

[9] "**Decoherence of electron beams by electromagnetic field fluctuations**," Y. Levinson *J. Phys. A* **37**, 3003 (2004).

[10] "**Thermal limitation of far-field matter-wave interference**," K. Hornberger *Phys. Rev. A* **73**, 52102 (2006).

[11] M. Brune, E. Hagley, J. Dreyer, X. Maitre,A. Maali, C. Wunderlich, J. M. Raimond, and S. Haroche, *Phys.Rev. Lett.***77**,4887(1996).

[12] C. Monroe, D. Meekhof, B. King and D. Wineland *Science* **272** 1131 (1998).

[13] "**Pointer states via decoherence in a quantum measurement** ", Anu Venugopalan, *Physical Review A* , **61**, 012102 (2000).

[14] "**Quantum coherence and interference of damped free particles**", C. M. Savage and D. F. Walls, *Phys. Rev. A* **32**, 3487 (1985).

[15] M. A. Neilson and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, UK 2000).

[16] "**Loss of coherence in double-slit diffraction experiments**", A.S. Sanz, F. Borondo and M.J. Bastiaans, [*Phys. Rev. A***71**, 042103 (2005).

[17] "**Interference of mesoscopic particles: quantum-classical transition**", P. Facchi, S. Pascazio and T. Yoneda, [*quant-ph/0509032*].

[18] "**Path integral approach to quantum brownian motion**, A. O. Caldeira and A. J. Leggett, *Physica A* **121**, 587 (1983).

[19] "**Environment-induced decoherence I: The Stern-Gerlach measurement**", A. Venugopalan, D. Kumar, R. Ghosh, *Physica* **220**, (1995).

[20] "**Brownian motion of a quantum particle"**, D. Kumar, Phys. Rev. A **29**, 1571 (1984).

[21] **"Theory of decoherence in matter-wave Talbot-Lau interferometer"** K. Hornberger, John E. Sipe and Markus Arndt, *Phys. Rev. A*, 70, 053608 (2004).